Origin of magnetic moment

electron spin $\vec{S}$

electron orbital angular momentum $\vec{L}$

$$\vec{M}(\vec{H}) = -\frac{1}{V}\frac{\partial}{\partial H}E_0(H)$$ magnetization density

$E_0(H)$ is ground state energy.

$$\chi = \frac{\partial|\vec{M}|}{\partial|\vec{H}|}$$ magnetic susceptibility

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Diamagnetism (in insulators)

$$\underline{\chi} = \frac{\partial M}{\partial H} \simeq -\frac{1}{V}\frac{\partial^2 E_0(H)}{\partial H^2}$$

$$E = PV + MH + k_B T + \cdots$$

Change in $\vec{L}$ → Lenz's law diamagnetism

Hydrogen atom, $1e^-$ 1S state, $\vec{L}=0$

Helium atom, $2e^-$ $1S^2$, $\vec{S}=\vec{L}=0$

filled e^- shell ″

superconductor → Meissener effect

Langevin Diamagnetic Eq.

(classical case)

Larmor theorem

e^- pression in $\vec{H}$, $\omega = \frac{eB}{2mc}$

$$\mu = I \cdot A$$
$$= \pi \rho^2 \cdot \left(-\frac{Ze}{c}\right)\left(\frac{1}{2\pi}\frac{eB}{2mc}\right)$$

$$= -\frac{Ze^2 B}{4mc^2}\langle \rho^2 \rangle, \quad \langle \rho^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle$$
$$\langle r^2 \rangle = \langle x^2 \rangle + \langle y^2 \rangle + \langle z^2 \rangle$$
$$\langle \rho^2 \rangle = \frac{2}{3}\langle r^2 \rangle$$
$$\chi = \frac{N\mu}{B} = -\frac{NZe^2}{6mc^2}\langle r^2 \rangle$$

Quantum theory (mononeulear system)

$\vec{\nabla} \cdot \vec{B} = 0$ Maxwell's eqs.

$$\vec{B} \equiv \vec{\nabla} \times \vec{A}$$

Halmiltonian

$$\mathcal{H} = \frac{1}{2m}\left(\vec{p} - \frac{e}{c}\vec{A}\right)^2 + Q\varphi$$
$$= \underbrace{\frac{p^2}{2m} + Q\varphi}_{\mathcal{H}_0} + \frac{ie\hbar}{2mc}\left(\vec{\nabla} \cdot \vec{A} + \vec{A} \cdot \vec{\nabla}\right) + \frac{e^2}{2mc^2}A^2$$

We choose $\vec{B} = B\hat{z}$

e.g. $\vec{A} = -\frac{1}{2}\vec{r} \times \vec{B}$ $\begin{cases} A_x = -\frac{1}{2}yB \\ A_y = \frac{1}{2}xB \\ A_z = 0 \end{cases}$

$$\mathcal{H} = \mathcal{H}_0 + \Delta T + \Delta\mathcal{H}_{spin}$$

$$\Delta \mathcal{J} = \frac{ieh\nu}{2mc}\left(x\frac{\partial}{\partial y} - y\frac{\partial}{\partial x}\right) + \frac{e^2\nu^2}{8mc^2}(x^2+y^2)$$

$$\langle \rho^2\rangle = \frac{2}{3}\langle r^2\rangle$$

For spin $\Delta \mathcal{H}_{spin} = g_0 \mu_B B S_z$

$g_0 \approx 2$ for e^-

$$S_z = \sum_i S_z^i$$

$$\Delta \mathcal{J} = \mu_B \vec{L}\cdot\vec{B} + \frac{e^2B^2}{8mc^2}(x^2+y^2)$$

$$\hbar\vec{L} = \vec{r}_i \times \vec{p}_i$$

$$\mathcal{H} = \mathcal{H}_0 + \mu_B(\vec{L} + g_0\vec{S})\cdot\vec{B} + \frac{e^2B^2}{8mc^2}(x^2+y^2)$$

$\langle\rho^2\rangle = \frac{2}{3}\langle r^2\rangle$

① ground state of a system with filled e^- shells

$$J|0\rangle = L|0\rangle = S|0\rangle = 0$$

$$\Delta E_0 = \frac{e^2B^2}{12mc^2}B^2\langle 0|\sum_i r_i^2|0\rangle$$

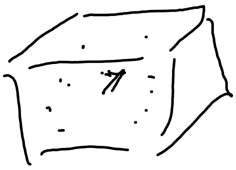
$$\chi = -\frac{N}{V}\frac{\partial^2 F_0}{\partial B^2} = -\frac{e^2}{6mc^2}\frac{N}{V}\underbrace{\langle 0|\sum_i r_i^2|0\rangle}_{>0}$$

For diamagnetic materials $\chi < 0$

Classical theory of paramagnetism

(super paramagnetism)

A volumn of N identicle magnetic moments μ



$$E = -\vec{\mu}\cdot\vec{B} = -\mu B \cos\theta$$

Boltzmann factor

$$e^{-\frac{U}{k_BT}} = e^{\frac{\mu B}{k_BT}\cos\theta}$$

$$p(\theta)\,d\theta = \frac{e^{\frac{\mu B}{kT}\cos\theta}\sin\theta\, d\theta}{\int_0^{\pi} e^{\frac{\mu B}{k_B T}\cos\theta}\sin\theta\, d\theta}$$

$$M = N\mu\,\overline{\cos\theta}$$

$$= N\mu \int_0^{\pi} \cos\theta\; p(\theta)\,d\theta$$

$$= N\mu\,\frac{\int_0^{\pi} \exp\left(\frac{\mu B}{kT}\cos\theta\right)\cos\theta\sin\theta\, d\theta}{\int_0^{\pi} \exp\left(\frac{\mu B}{kT}\cos\theta\right)\sin\theta\, d\theta}$$

$$\frac{\mu B}{kT} \equiv \alpha\,, \quad \cos\theta \equiv x\,, \quad \frac{dx}{d\theta} = -\sin\theta$$

$$dx = -\sin\theta\, d\theta$$

$$M = N\mu\,\frac{\int_{-1}^{1} e^{\alpha x}\, x\, dx}{\int_{-1}^{1} e^{\alpha x}\, dx}$$

$$\int_{-1}^{1} e^{\alpha x} dx = \frac{1}{\alpha}\left[e^{\alpha x}\right]_{-1}^{1} = \frac{1}{\alpha}\left(e^{\alpha} - e^{-\alpha}\right)$$

$$\frac{d}{d\alpha} \Rightarrow \int_{-1}^{1} e^{\alpha x}\, x\, dx = \frac{1}{\alpha}\left(e^{\alpha} + e^{-\alpha}\right) - \frac{1}{\alpha^2}\left(e^{\alpha} - e^{-\alpha}\right)$$

$$M = N\mu\left(\frac{e^{\alpha} + e^{-\alpha}}{e^{\alpha} - e^{-\alpha}} - \frac{1}{\alpha}\right)$$

$$= N\mu\,\underbrace{\left(\coth\alpha - \frac{1}{\alpha}\right)}$$

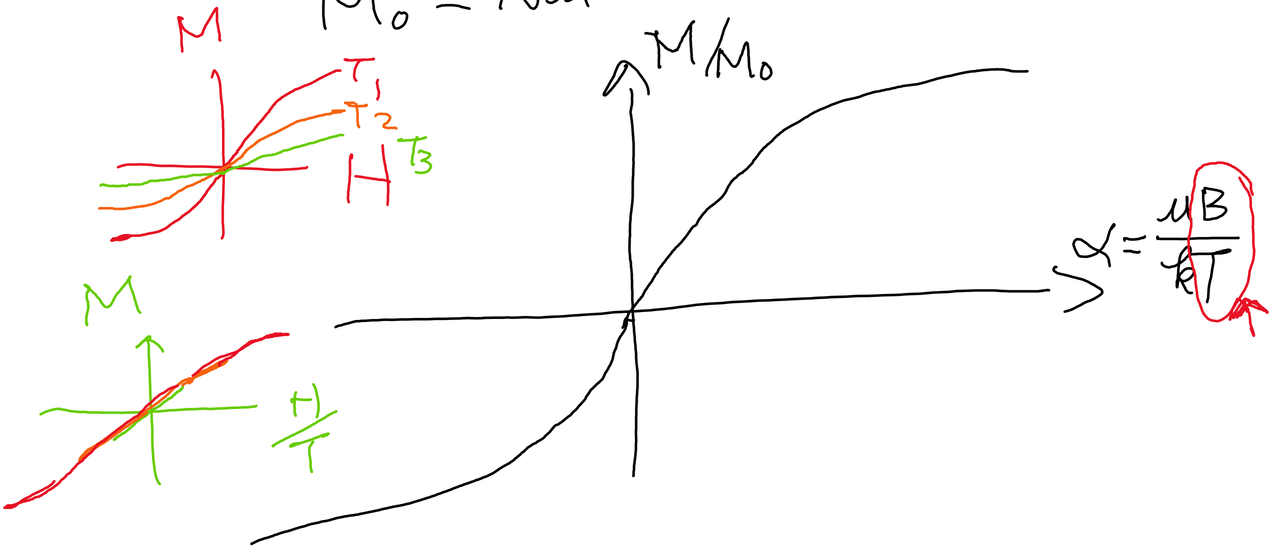
Langevin function

$$L(\alpha) = \frac{\alpha}{3} - \frac{\alpha^3}{45} + \frac{2\alpha^5}{945} - \cdots$$

when $\alpha < 0.5$, $L(\alpha) \approx \frac{\alpha}{3}$

$\alpha << 1$, $L(\alpha) \to 1$

$M_0 = N\mu$



For small $\alpha = \frac{\mu B}{kT}$ small B / high T

$$M = \frac{N\mu\alpha}{3} = \frac{N\mu^2 B}{3k_B T}$$

$$\chi = \frac{M}{H} = \frac{N\mu^2}{3k_B}\frac{1}{T} = \frac{C}{T}$$

Curie's law

with Curie's constant

$$C \equiv \frac{N\mu^2}{3k_B}$$

This is based on non-interacting μ's

Quantum Theory of paramagnetism in free space

$$\mu = \gamma\hbar\vec{J} = -g\mu_B\vec{J} \quad \text{for atom. ion}$$

$$\hbar\vec{J} = \hbar\vec{L} + \hbar\vec{S}$$

γ : gyromagnetic ratio
magnetogyric ratio

g factor. spectroscopic splitting factor

$$g\mu_B \equiv -\gamma\hbar$$

$$g = 2.0023 \quad \text{for free } e^-$$

$$g = 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)}$$

Landé equation for free atom

$$\mu_B \equiv \frac{e\hbar}{2mc} \quad \text{in cgs}$$

Bohr magneton

$$U = -\vec{\mu}\cdot\vec{B} = m_J g\mu_B B$$

$$m_J = -J, -J+1, \cdots, +J$$

For an e^-

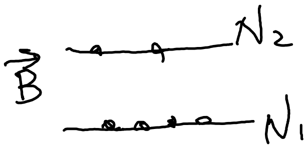
$$\begin{cases} \vec{\mu}_{orbit} = \dfrac{eh}{4\pi mc} = \dfrac{e}{2mc}\dfrac{h}{2\pi} = -\dfrac{e}{2mc}\vec{P}_{orbit} \\ \vec{\mu}_{spin} = \dfrac{eh}{4\pi mc} = \dfrac{e}{mc}\dfrac{1}{2}\dfrac{h}{2\pi} = -\dfrac{e}{mc}\vec{P}_{spin} \end{cases}$$

$g = 1$ for orbital moment
$g = 2$ for spin

for a single spin, no orbital motion

$m_J = \pm\frac{1}{2}$. $g = 2$

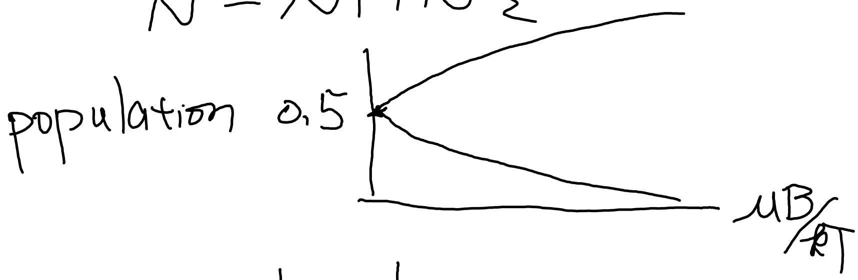
Consider a two-level system at equilibrium



$$\frac{N_1}{N} = \frac{\exp\left(\frac{\mu B}{kT}\right)}{\exp\left(\frac{\mu B}{kT}\right) + \exp\left(\frac{-\mu B}{kT}\right)}$$

$$\frac{N_2}{N} = \frac{\exp\left(-\frac{\mu B}{kT}\right)}{\quad}$$

$$N = N_1 + N_2$$



$$M = (N_1 - N_2)\mu$$

$$= N\mu \cdot \frac{e^{x} - e^{-x}}{e^{x} + e^{-x}} \qquad x \equiv \frac{\mu B}{kT}$$

$$= N\mu \tanh x$$

for $x \ll 1$, $\tanh x = x$

$$M \hat{=} \frac{N\mu^2 B}{kT}$$

An atom with quantum number J

$$U = -m_J g \mu_B B$$

Boltzmann factor $e^{-\frac{U}{k_B T}} = e^{m_J g \mu B / kT}$

$$M = N \frac{\sum_{m_J} m_J g \mu \exp[m_J g \mu B / kT]}{\sum \exp[m_J g \mu B / kT]}$$

$\Sigma_{m_J} \exp[m_J g \mu B / kT]$

$$x \equiv \frac{gJ\mu B}{kT}$$

$$= N g J \mu \left[\frac{2J+1}{2J} \coth\left(\frac{2J+1}{2J} x\right) - \frac{1}{2J} \coth\left(\frac{x}{2J}\right) \right]$$

$$= N g J \mu B_J(x)$$

Brillouin function

Curie – Brillouin's law

$$\frac{M}{NgJ\mu} = B_J(x)$$

for $x \ll 1$ $\quad \coth x = \frac{1}{x} + \frac{x}{3} - \frac{x^3}{45} + \cdots$

$J \to \infty$

$$B_J(x) = \coth x - \frac{1}{2J} \cdot \frac{2J}{x} = \coth x - \frac{1}{x}$$

B_J goes to Langevin func.

$$\chi = \frac{M}{B} \simeq \frac{NJ(J+1)g^2\mu^2}{3kT} \simeq \frac{NP^2\mu^2}{3kT}$$

$P \equiv g[J(J+1)]^{1/2}$

effective number of Bohr magnetons

$$= \frac{C}{T}$$

Compared with experimental results

agrees well with trivalent Lanthanide ions! (rare earth)

except Eu^{3+}, Sm^{3+}, high state L-S

$\Longrightarrow$ Iron group ions the agreements are poor.

agrees better with

$$p = 2[S(S+1)]^{1/2} \Rightarrow L$$

$\because$ 4f shell is deep inside the ions,

3d shell is the outmost shell

inhomogeneous crystal field is important!

L·S coupling broke, J no longer a good Quantum numb

Quenching of the orbital angular momen L_z.

Consider in a free atom,

there is one e^- in p-state

degenerate states

$$\begin{cases} R_{nl}(r)\,Y_{11}(\theta,\varphi) & \propto f(r)\left[\dfrac{x+iy}{\sqrt{2}}\right] \\ R_{nl}(r)\,Y_{10}(\theta,\varphi) & \propto f(r)\,z \\ R_{nl}(r)\,Y_{1,-1}(\theta,\varphi) & \propto f(r)\left[\dfrac{x-iy}{\sqrt{2}}\right] \end{cases}$$

e^-, $L=1$ orbital

take $S=0$

in a crystal with orthorhombic symmetry

$$e\varphi = Ax^2 + By^2 - (A+B)z^2, \quad A, B \text{ const.}$$

$\nabla^2 \varphi = 0$

$U_x = \underline{x} f(r), \quad U_y = \underline{y} f(r), \quad U_z = z f(r)$

$\mathcal{L}^2 U_i = L(L+1) U_i = 2 U_i \quad , \; i = x, y, z.$

angular momentum operator

U_i's are diagonal w.r.t $e\varphi$

Check the off-diagonal terms are 0.

$\langle U_x | e\varphi | U_y \rangle = \langle U_y | e\varphi | U_z \rangle = \langle U_z | e\varphi | U_x \rangle = 0$

$\langle U_x | e\varphi | U_y \rangle = \int xy |f(r)|^2 \{ A x^2 + B y^2 - (A+B) z^2 \} \, dx dy dz$

odd in x, y. $\Rightarrow = 0$

orbital momentum

$\langle U_x | L_z | U_x \rangle = 0 = \langle U_y | L_z | U_y \rangle$
$= \langle U_z | L_z | U_z \rangle$

$|1\rangle = \frac{x+iy}{\sqrt{2}}, \quad |0\rangle = z, \quad |-1\rangle = \frac{x-iy}{\sqrt{2}}$

$\longrightarrow x = \frac{1}{\sqrt{2}} (|1\rangle + |-1\rangle), \quad y = \frac{1}{i\sqrt{2}} (|1\rangle - |-1\rangle)$

$\hookrightarrow \langle x | L_z | x \rangle, \quad \langle y | L_z | y \rangle, \quad \langle z | L_z | z \rangle$

Magnetic state $\quad {}^{2S+1}L_J$

$L: \quad 0 \quad 1 \quad 2 \quad 3 \quad 4$

S P D F G

e.g. full shells, $L=0$, $S=0$, $\Rightarrow J=0$

Gd^{3+} $4f^7 \cancel{5s^2} \cancel{5p^6}$ $n=4, l=3$

half full shell, $L=0$, $S=\frac{1}{2}(2l+1)$

$L=0$, $S=7/2 \Rightarrow J=L+S=7/2$

$^8S_{7/2}$

Ferromagnetism

Exchange coupling ※

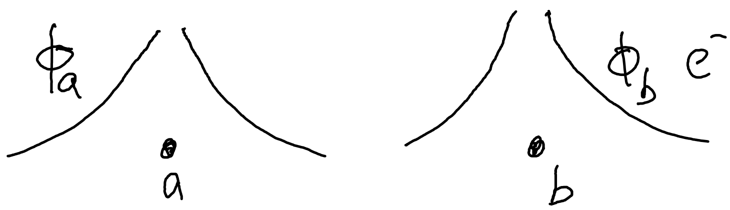
microscopically

ferro $\Leftrightarrow$ antiferro $\Longleftrightarrow$ ferri

$\uparrow\uparrow\uparrow\uparrow\uparrow$ $\uparrow\downarrow\uparrow\downarrow\uparrow$ + $\uparrow\downarrow\uparrow\downarrow\uparrow$

$\uparrow\downarrow\uparrow\downarrow\uparrow$

Example: $2e^-$ system H_2



$$H\Psi = \left[\sum_{i=1}^{2} -\frac{\hbar^2}{2m}\nabla_i^2 - \frac{e^2}{|\vec{r}-\vec{R}|} - \frac{e^2}{|\vec{r}-\vec{R}|}\right]\Psi(\vec{r}_1,\vec{r}_2)$$

$|\vec{r}_i - R_a| \quad |r_i - R_b|$

$= E \Psi$

spin-independent Schrödinger equation can lead to magnetization

spin states :

$|\uparrow\uparrow\rangle, \ |\uparrow\downarrow\rangle, \ |\downarrow\uparrow\rangle, \ |\downarrow\downarrow\rangle$

$\psi_s \ \overset{\frown}{\uparrow\uparrow} \xrightarrow{\text{if}} -\psi_s \qquad \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

exchange spins

$S=0, S_z=0$, singlet state

$\psi_s \ \overset{\frown}{\uparrow\uparrow} \rightarrow +\psi_s \quad |\uparrow\uparrow\rangle \qquad S=1, S_z=+1$

$\frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle) \quad S=1, S_z=0$

$|\downarrow\downarrow\rangle \qquad S=1, S_z=-1$

triplet state

Pauli exclusion principle

Ψ must chang sign when particles are exchanged

(spin) singlet $S=0$, antisym $\psi_s \oplus$ sym. ψ

triplet $S=1$, sym $\psi_s \oplus$ antisym ψ

$2e^-$ system

$E_s < E_t$

$h\varphi(\vec{r}) = \varepsilon \varphi(\vec{r})$ Sch. eq. for $1e^-$

with solutions $\varphi_0, \varphi_1, \cdots \ \varepsilon_0, \varepsilon_1, \cdots$

Two e^- solution

singlet state, sym $\psi(\vec{r}_1, \vec{r}_2) \approx \varphi_0(\vec{r}_1)\varphi_0(\vec{r}_2)$

$E_s = 2\varepsilon_0$

triplet state, antisym.

$$\psi(\vec{r}_1, \vec{r}_2) = \frac{1}{\sqrt{2}}\left[\varphi_0(\vec{r}_1)\varphi_1(\vec{r}_2) - \varphi_0(\vec{r}_2)\varphi_1(\vec{r}_1)\right]$$

$$E_t = \varepsilon_0 + \varepsilon_1$$

$$\therefore E_s - E_t = \varepsilon_0 - \varepsilon_1 < 0,$$ singlet-triplet splitting

If $\varphi_0(r) = \phi_a(r) + \phi_b(r)$ sym

$\varphi_1(r) = \phi_a(r) - \phi_b(r)$ antisym

ignore $e^- - e^-$ interaction

$$J = E_s - E_t \neq 0$$

Heisenberg model

$$E = \frac{1}{2}\sum J_{ij}\, \vec{S}_i \cdot \vec{S}_j$$

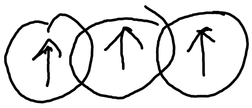
for $J < 0$ singlet ground state $S=0$ nonmagnetic

$J > 0$ ground state with $S \neq 0$, magnetic

Exchang interaction is the source of magnetization

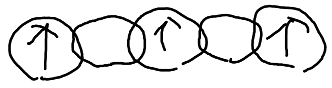
Types of exchange interaction.

① direct exchange



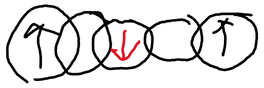
overlap between magnetic ions

② Super exchange



same magnetic ions overlap with non-magnetic ions

③ double exchange



different magnetic ions states (Mn^{3+}, Mn^{4+}) overlap with non-magnetic ions

④ indirect exchange

mediated by conduction e's

⑤ itinerant exchange

between conduction e's

Map the hydrogen molecule into a spin system with 4 eigenstates

spin operator

$$\vec{S}_i^{\,2} = s(s+1) = \tfrac{1}{2}\left(\tfrac{1}{2}+1\right) = \tfrac{3}{4} \text{ for individual spin}$$

$$\vec{S}^{\,2} = (\vec{S}_1 + \vec{S}_2)^2$$

$$= \vec{S}_1^{\,2} + \vec{S}_2^{\,2} + 2\vec{S}_1\cdot\vec{S}_2$$

$$= \frac{3}{4} + \frac{3}{4} + 2\vec{S}_1\cdot\vec{S}_2$$

For singlet state, $S=0$

$$\vec{S}_1\cdot\vec{S}_2 |\psi_s\rangle = \frac{1}{2}\left[\vec{S}^{\,2} - \vec{S}_1^{\,2} - \vec{S}_2^{\,2}\right]|\psi_s\rangle$$

$$= \frac{1}{2}\left[0(0+1) - \frac{3}{4} - \frac{3}{4}\right]|\psi_s\rangle$$

$$= -\frac{3}{4}|\psi_s\rangle$$

For triplet state, $S=1$.

$$\vec{S}_1\cdot\vec{S}_2|\psi_t\rangle=\frac{1}{2}\left[1(1+1)-\frac{3}{4}-\frac{3}{4}\right]|\psi_t\rangle$$
$$=+\frac{1}{4}|\psi_t\rangle$$

Take a trial spin Hamiltonian

$$H^{spin}=\frac{1}{4}(E_s+3E_t)-\underbrace{(E_s-E_t)}_{J}\vec{S}_1\cdot\vec{S}_2$$

$$H^{spin}|\psi_s\rangle=E_s|\psi_s\rangle$$
$$H^{spin}|\psi_t\rangle=E_t|\psi_t\rangle$$

redefine zero energy

$$\Rightarrow H^{spin}=-J\,\vec{S}_1\cdot\vec{S}_2\ ,\quad J\equiv E_s-E_t$$

For a N-ion System

$$\boxed{H^{spin}=-\sum_{i<j}J_{ij}\,\vec{S}_i\cdot\vec{S}_j}$$

Heisenberg Spin Hamiltonian

J_{ij} : exchange coupling constant

if $J_{ij}>0$ ferromagnets

$S_i \parallel S_j$

$J_{ij}<0$ $\quad \vec{S}_i\cdot\vec{S}_j<0 \Rightarrow$ antiferro

Ising model

$$H^{Ising}=-\sum J_{ij}\,S_iS_j$$

$S_{i,j} = \pm 1$

Molecular-field approximation

$$H^{spin} = -\sum_{i<j} J_{ij} \vec{S}_i \cdot \vec{S}_j$$

$$\simeq -\sum_k \left[\sum_{i<k} J_{ik} \langle \vec{S}_i \rangle \cdot \vec{S}_k + \sum_{k<j} J_{kj} \vec{S}_k \cdot \langle \vec{S}_j \rangle \right]$$

$$= -\sum_k \vec{S}_k \cdot \left(\sum_{i \neq k} J_{ik} \langle \vec{S}_i \rangle \right)$$

$$H^{spin} = -\sum_i \vec{\mu}_i \cdot \vec{B}_i$$

$\vec{\mu}_i = g\mu_0 \vec{S}_i$ magnetization of ion i

$\vec{B}_i$: average magnetic field at the position of ion i

considering spin along z-axis

$$H^{spin} = -\sum_i (g\mu_0 S_i^z) \cdot B_i^z = -\sum_i \mu_i^z B_i^z$$

$$B_i^z = \frac{1}{g\mu_0} \sum_{i \neq j} J_{ij} \langle S_j^z \rangle$$

molecular field

Weiss field

$$\vec{B}_i = \lambda \vec{M}$$

$$B_i^z = \lambda M^z = \lambda g\mu_0 \langle S_j^z \rangle / V$$

M^z : magnetization per unit volumn

λ : mean field coefficient

Under external field $\vec{B}$ along $\hat{z}$

$$H^{tot} = -\sum_i \mu_i^z (B_i^z + B^z)$$

resultant $M^z = \sum_i \mu_i^z = N \langle \mu^z \rangle$

$$\langle \mu_z \rangle = \mu_0 \tanh \frac{(B_i^z + B^z)\mu_0}{k_B T}$$

two-level system

a). external field $B^z = 0$

$$M^z = N \langle \mu^z \rangle = N\mu_0 \tanh \frac{\mu_0 B^z}{k_B T}$$

$$\approx \frac{N \mu_0^2 B^z}{k_B T}$$

$$\chi = \frac{M}{B} = \frac{N \mu_0^2}{k_B T}$$

$$\langle \mu^2 \rangle = \underbrace{g^2}_{=4} \mu_0^2 \underbrace{\langle \vec{S}^2 \rangle}_{=\frac{3}{4}} = 3\mu_0^2$$

$$\chi = \frac{N \langle \mu^2 \rangle}{3 k_B T}$$ Weiss law for paramagnetic suscep

b) $B_i^z \neq 0$

$$M^z = N\mu_0 \tanh \frac{(B^z + \lambda M^z)\mu_0}{k_B T}$$

$\left(x \equiv \frac{aB}{k_B T} \ll 1 \right)$ For $M^z \to 0$, $B^z \to 0$, or $T \to \infty$

$$M^z = \frac{N \mu_0^2 (B^z + \lambda M^z)}{k_B T}$$

$$M^z\left(1-\frac{N\mu_0^2\lambda}{k_BT}\right)=\frac{N\mu_0^2B^z}{k_BT}$$

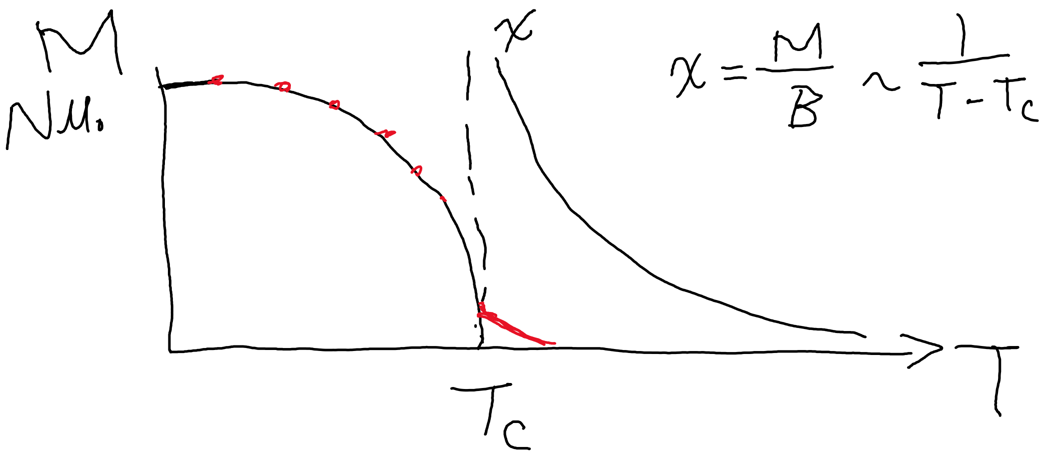
$$\chi=\frac{M^z}{B^z}=\frac{N\mu_0^2}{k_BT\left(1-\frac{N\mu_0^2\lambda}{k_BT}\right)}$$

$$=\frac{N\mu_0^2}{k_BT-N\mu_0^2\lambda}$$

$$=\frac{N\langle\mu^2\rangle/3}{k_BT-N\langle\mu^2\rangle\lambda/3}$$

$$=\frac{N\langle\mu^2\rangle/3k_B}{T-T_c},\quad T_c=\frac{N\langle\mu^2\rangle\lambda}{3k_B}$$

Curie-Weiss law

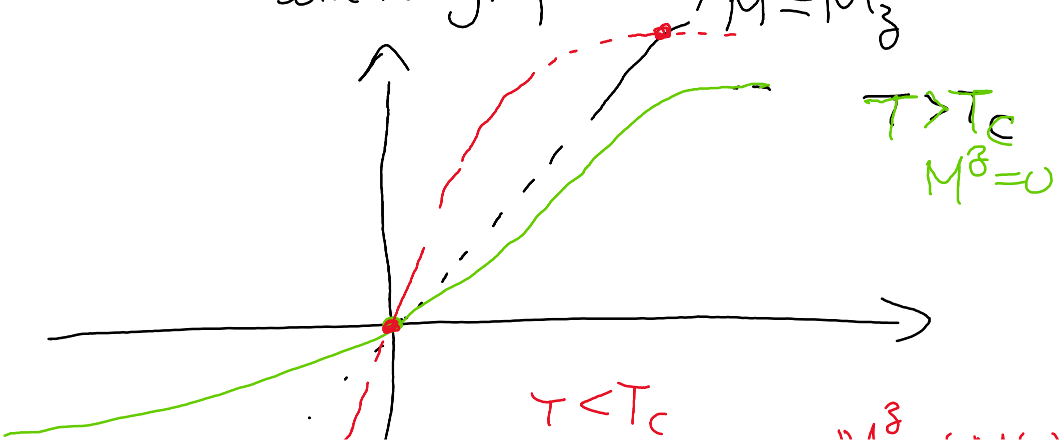


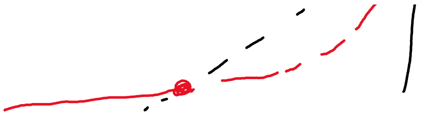
$$\chi=\frac{M}{B}\sim\frac{1}{T-T_c}$$

$T<T_c$, $B^z\to 0$

$$M^z=N\mu_0\tanh\frac{\lambda\mu_0M^z}{k_BT}$$

solve it graphically

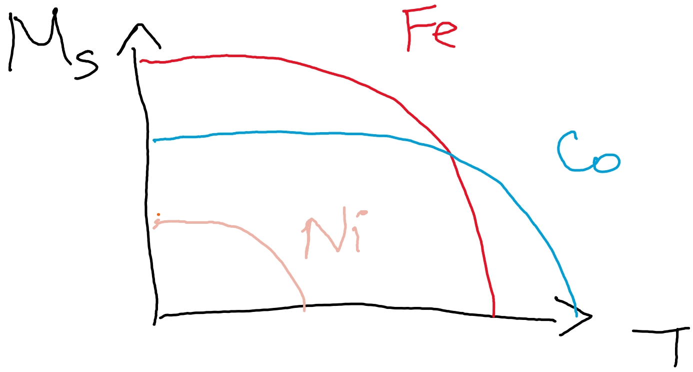




$N\mu_s \tanh[\ \ ]$ $M = \pm M(T), 0$

$M \sim (T-T_c)^{1/2}$ from mean field theory

$\chi \sim (T-T_c)^{-1}$



Superparamagnetic limit for magnetic storage media?

Néel relaxation time τ_N ($H=0$, T_{room})

Néel-Arrhenius equation

$$\tau_N = \tau_0 \exp\left(\frac{KV}{k_B T}\right)$$

due to thermal fluction

$\frac{1}{\tau_0}$, atteemp frequency, f_0

K: magnetic anisotropy energy density

$$-\frac{dM_r}{dt} = f_0 M_r \exp\left(-\frac{KV}{k_B T}\right) = \frac{M_r}{\tau}$$

$$M_r(t) = M_{r0} \exp\left(-\frac{t}{\tau}\right)$$

$$\tau^{-1} = f_0 \exp\left(-\frac{KV}{k_B T}\right)$$

if we finish one measurement

[illegible]

in 100 sec.

$$0.01 = 10^{9} \exp\left(-\frac{KV}{k_B T}\right)$$

$$V_p = \frac{25\, k_B T}{K}$$

if one wants to retain the data for 10 years

$$86400 \times 365 \times 10 \sim 3.15 \times 10^{8}$$

$$\frac{1}{3.15 \times 10^{8}} = 10^{9} \exp\left(-\frac{KV}{k_B T}\right)$$

magnetic bit $\quad V = \frac{40\, k_B T}{K}$